Q- Three particles, each with a mass of 0.25 kg, are located at (- 4.0 m, 0), (2.0 m, 0), and (0, 3.0 m) and are acted on by forces F_1 = 3 N along negative y-direction, F_2 = 5 N along positive y-direction and F_3 = 4 N along x direction respectively. Find position of center of mass of the system and acceleration (magnitude and direction) of the center of mass of the system.

The x coordinate of position of center of mass of the system is given by

$$x_{cm} = \frac{\sum mx}{\sum m} = \frac{mx_1+mx_2+mx_3}{m+m+m}$$

Or $$x_{cm} = \frac{-4.0+2.0+0}{3} = -\frac{2}{3}$$

And similarly, y coordinate is given by

$$y_{cm} = \frac{\sum my}{\sum m} = \frac{my_1+my_2+my_3}{m+m+m}$$

Or $$y_{cm} = \frac{0+0+3}{3} = 1$$

Hence the position of the center of mass of the system is (-2/3 m, 1m)

Now the center of mass of a system is the point for which Newton's laws hold good whatever be the position of the force applied. The acceleration of center of mass depends on the force and the total mass of the system as

$$\vec{a}_{CM} = \frac{\vec{F}}{M_{total}}$$

Now the resultant of the three forces is given by (in N)

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

Or $$\vec{F} = -3.0\,\hat{y} + 5.0\,\hat{y} + 4.0\,\hat{x}$$ [Here $\hat{x}$ and $\hat{y}$ are unit vectors in corresponding directions]

Or $$\vec{F} = 4.0\,\hat{x} + 2.0\,\hat{y}$$

Thus, the acceleration of center of mass of the system of particles is given by

$$\vec{a}_{CM} = \frac{\vec{F}}{M_{total}} = \frac{4.0\,\hat{x}+2.0\,\hat{y}}{0.25+0.25+0.25} = 5.33\,\hat{x} + 2.67\,\hat{y}$$

Or $$\vec{a}_{CM} = 5.33\,\hat{x} + 2.67\,\hat{y}$$

Thus, the magnitude of the acceleration of center of mass will be

$$|\vec{a}_{CM}| = \sqrt{a_x^2 + a_y^2} = \sqrt{5.33^2 + 2.67^2} = 5.96\ m/s2$$

And its direction is given by

$$tan\theta = \frac{a_y}{a_x}$$

Or $$tan\theta = \frac{2.66}{5.33} = 0.50$$

Or $$\theta = tan^{-1}0.50 = 26.6^0$$

Hence the acceleration of center of mass is **5.96 m/s²** in the direction making angle **26.6 degree** with the positive direction of x axis.