Q- A 3.00 kg particle has a velocity of (3.00i - 4.00J) m/s.
(a) Find its x and y components of momentum.
(b) Find the magnitude and direction of its momentum.

Reading:
As a vector is having no effect in a direction perpendicular to it we can resolve (break) it in three mutually perpendicular directions called x, y, and z directions and the three parts are called the components. Each vector is expressed by magnitude and direction, the components are shown by the product of their magnitude and a vector in the respective direction has unit magnitude called unit vector. $\hat{i}, \hat{j}$ and $\hat{k}$ denotes unit vectors in x, y and z direction respectively. In case of motion in a plane, components are taken in only two mutually perpendicular directions and component in the third direction will be zero.

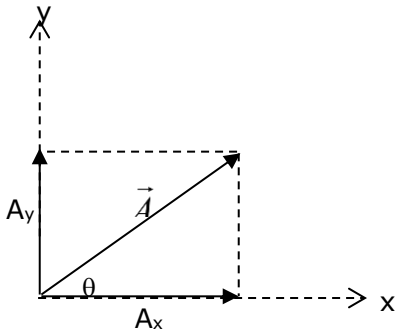
A vector $\vec{A}$ (in coplanar situation) is written in component form as

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

Here A_x and A_y are the magnitudes of the components in x and y directions respectively.

As the two component vectors are perpendicular to each other hence their magnitude and the magnitude of their resultant $\vec{A}$ is related by

$$A = \sqrt{A_x^{\,2} + A_y^{\,2}} \qquad \text{---------- (1)}$$



And if $\vec{A}$ is making an angle θ with x direction then we have the relation

$$\tan\theta = \frac{A_y}{A_x} \qquad \text{---------- (2)}$$

Solution:

(a) Momentum of a body is the effect the moving bodies can produces (called quantity of motion) which depends on the mass of the body and its velocity and hence measured by the product of mass of the body and its velocity. Thus, if a body of mass m is moving with velocity $\vec{v}$, then its momentum is given by

$$\vec{P} = m * \vec{v}$$

Hence the momentum of the particle is given by

$$\vec{P} = m * \vec{v} = 3..00 * \left(3.00\hat{i} - 4.00\,\hat{j}\right) \text{ kg m/s}$$

Or $\vec{P} = \left(9.00\hat{i} - 12.00\,\hat{j}\right)$ kg m/s

Hence the magnitude of x component of the momentum of the particle is 9.00 kg m/s and y component will be -12.00 kg m/s. (negative means negative y direction)

(b) Find the magnitude and direction of its momentum.

The momentum is given by

$$\vec{P} = \left(9.00\hat{i} - 12.00\hat{j}\right) \text{ kg m/s}$$

Hence its magnitude is given by (equation 1)

$$P = \sqrt{P_x^2 + P_y^2} = \sqrt{(9.00)^2 + (-12.00)^2} = \sqrt{81.00 + 144.00} = 15.00 \text{ Kg m/s.}$$

And the direction is given by

$$\tan\theta = \frac{P_y}{P_x} = \frac{-12.00}{9.00} = -1.33$$

Gives $\theta = \tan^{-1}(-1.33) = -53^0$

The negative sign shows that the direction of P is in fourth quadrant and hence the angle with x axis in positive (anticlockwise) direction will be

$\theta = 360^0 - 53^0 = 307^0$

The answer is (a) (9.00i - 12.0j) kg m/s. (b) 15.0kg m/s at 307 degrees.