Q- A spherical brass shell of mass 2.50 kg barely floats in water such that it is completely submerged with the waterline just at its top. The inside of the shell is completely evacuated. The density of brass is 8600 kg/m^3 and that of water is 1000 kg/m^3.

(a) calculate the outer radius of the shell R_2

As the shell is just floating inside water it is in equilibrium position. The forces acting on it are its weight and the upthrust of water, both are equal in magnitude and opposite in direction. Thus the upthrust is equal to the weight of the shell and is equal to the weight of the water displaced.

Upthrust of water = weight of the shell

Or weight of the water displaced = weight of the shell

Or volume of water displaced * density of water *g = mass of the shell*g

Or $\frac{4}{3}\pi R_2^3 * 1000 = 2.5$

Or $R_2^3 = \frac{3}{4\pi * 1000} * 2.5$

Or $R_2^3 = \frac{3}{4*3.14*1000} * 2.5$

Or $R_2^3 = \frac{3}{4*3.14*1000} * 2.5$

Or $R_2^3 = 5.97 * 10^{-4}$

Gives R_2 = 0.0842 m

(b) calculate the inner radius R_1

As the density of a substance is mass/volume, we get

Volume = mass/ density

Now volume of the brass in the shell is equal to the volume of the outer sphere minus volume of the inner sphere. Thus,

$\frac{4}{3}\pi R_2^3 - \frac{4}{3}\pi R_1^3 = \frac{2.5}{8600}$

Or $R_2^3 - R_1^3 = \frac{3}{4\pi} * \frac{2.5}{8600}$

Or $5.97 * 10^{-4} - R_1^3 = \frac{3}{4*3.14} * \frac{2.5}{8600}$

Or $R_1^3 = 5.97 * 10^{-4} - \frac{3}{4*3.14} * \frac{2.5}{8600}$

Or $R_1^3 = 5.97 * 10^{-4} - 6.94 * 10^{-5} = 5.27 * 10^{-4}$

Gives R_1 = 0.0808 m

(c) If the inner cavity is filled with helium should the shell float higher, sink of remain in the place.

If the inner cavity is filled with any substance it will add to the mass and hence weight of the shell while the upthrust will remain same. This will result in a net downward force of the shell and it will sink in water.