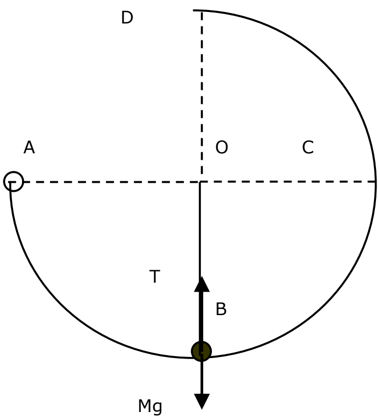
Q- A pendulum consists of a string of length L and a bob of mass M. The string is brought to a horizontal position and given the minimum initial speed enabling a pendulum to make a full turn in the vertical plane.
a) what is the maximum kinetic energy K of the bob?
b) what is the tension in the string when the kinetic energy is maximum?

Answer:

First of all we will discuss the condition for the bob to make full turn. For this when the bob reaches the heighest point the string should not get slacked. The forces acting on the bob are the weight Mg and the tension in the string T. Mg is always vertically downwards while the tension is twards the center O of the circular path. The resultant of the two must provide the necessary centriple force (Mv^2/L) to move the bob on the circular path. The speed of the bob is changing with the height and so will the required centripetal force, which decreases with the height. The tension changes in such a way that the tension and the component of weight along the center provide the centripetal force. As Mg is always downward, its maximum contribution to centripetal force is when it is at the top and and at that point the tension will be minimum, just zero. For that velocity of the bob at the heightest point D is given by

$Mg = Mv_D^2/L$
Or $v_D = \sqrt{(gL)}$ …………………(1) (v_D is velocity at D, the heighest point)

(If the initial velocity given at point A is less then a perticular value [$\sqrt{(3gL)}$] then the tension becomes zero before the bob reaching the top and the string get slacked and the bob leaves the circular path.)



a) Now the kinetic energy is maximum when the potential energy is minimum i.e. at the lowest point B. Using law of conservation of energy

total energy at B = total energy at D
$Mg \times 0 + (1/2)Mv_B^2 = Mg \times 2L + (1/2)Mv_D^2$
or KE at B: using eq (1)
$(1/2)Mv_B^2 = Mg \times 2L + (1/2)MgL$ = **(5/2)MgL**.

b) The tension at the lowest point is given by
$T_B - Mg = Mv_B^2/L$ ….(centripetal force)

Or $T_B = Mg + (5MgL)/L = Mg + 5Mg =$ **6Mg**.

(Initial speed of projection V_A is not required but canbe calculated using energy conservation at A and D. This may be downward or upward, not affacting the vel. at B or D.)