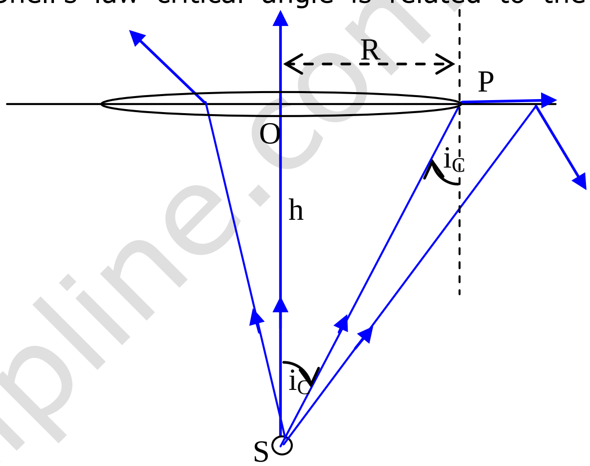
Q- A jewel thief hides a dimond by placing it at the bottom of a public swimming pool. He places a cicular raft on the surface of the water directly above the dimond as shown in the figure. The suftace of water is smooth and the pool is 2.0m deep. Find the miinimum diameter d of raft if it is to prevent the diamond from being seen by the people not in wager. (μ_{water} = 1.4)

The problem is based on the concept of total internal reflection.

When light passes through a denser medium to a rarer medium it moves away from the normal and hence angle of refraction is greater than the angle of incidence. With increase in angle of incidence angle of refraction increases and for an angle (less than 90deg.) of incidence the angle of refraction is just 90 deg. and if the angle of incidence is further increased the light will not get refracted to the rarer medium bet get reflected in the same denser medium this is called total internal reflection and the maximum angle for which refraction takes place is called critical angle. Using Snell's law critical angle is related to the refractive index as



$$_d\mu_r = \frac{\mu_r}{\mu_d} = \frac{\sin i}{\sin r} = \frac{\sin i_c}{sin90} = sini_c$$

if the rarer medium is air then for any denser medium we can write

$$\frac{1}{\mu} = \sin i_c \qquad \text{---------------} \ (1)$$

Now if the diamond (considered as point source) is placed in water light coming out has to go from the water to the air and hence the rays incident within the distance R from the point O on the surface, just above the source, such that the angle of incidence is just equal to critical angle can only emerge out of the surface. For all other rays at distance greater then R, the angle of incidence in the liquid will be greater than critical angle and cannot be refracted and get reflected inside the liquid back. Hence light will come out only from a circular area of the surface with radius R and that is to be blocked for the diamond to be invisible.

As shown in the figure the measure angle PSO will be equal to the critical angle and hence we have

$$\sin PSO = \sin i_C = OP/PS$$

Using Pythagoras theorem for right angle triangle PSO we have

$$\sin i_C = \frac{OP}{PS} = \frac{PO}{\sqrt{PO^2 + OS^2}} = \frac{R}{\sqrt{R^2 + h^2}}$$

Substituting in above equation from eq (1) we get

$$\frac{R}{\sqrt{R^2 + h^2}} = \frac{1}{\mu}$$

Or $\mu^2 R^2 = R^2 + h^2$

Or $R = \dfrac{h}{\sqrt{\mu^2 - 1}}$

This is the radius of the circular through which light will come out. Thus the area of this circular is given by

Hence the minimum diameter of the raft must be

$$d = 2R = \frac{2h}{\sqrt{\mu^2 - 1}}$$

or $d = \dfrac{2*2}{\sqrt{1.4^2 - 1}} = 4.08\, m$