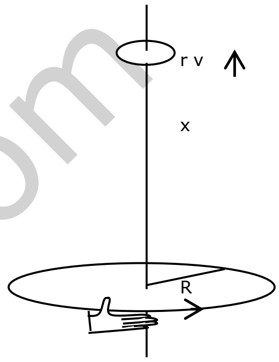
Q-Two parallel circular loops of wire having the same common axis. the smaller loop of (radius r) and is above the larger loop (radius R) by a distance X>>R. the magnetic field due to the counterclockwise current i in the larger loop is nearly uniform throughout the smaller loop. suppose that x is increasing at a constant rate dx/dt = v.
(a) Find the expression for the magnetic flux through the area of the smaller loop as a function of x.
(b) In the smaller loop, find the expression for the induced emf and the direction of the induced current.

Answer:
The current i in a circular loop create a magnetic field in the surrounding. The magnitude of this field, at a point on the axis of the loop, at a distance x is given by the expression

$$B_x = \frac{\mu_0 i R^2}{2(R^2+x^2)^{3/2}}$$



and its direction is given by right hand thumb rule, that is, if we put the right hand parallel to the loop (outside) such that the fingers point the direction of current, then the thumb (perpendicular to the fingers) shows the direction of the magnetic field. According to this law the field in our case is upward as in the figure.

The flux of this magnetic field passes through the smaller loop and as the surface of the loop is normal to the field direction, its area vector parallel to the field, the flux is given by

$$\varphi = \vec{B}\cdot\vec{A} = BA = \frac{\mu_0 i R^2 \pi r^2}{2(R^2+x^2)^{3/2}} \qquad \text{--------- (a)}$$

As the loop moves away, x changes and so the flux through the loop changes, induces an emf in the loop whose magnitude is given by using Faraday's law as

$$e = -\frac{d\varphi}{dt}$$ (Here –ve sign is according to Lenz law.)

So the induced emf in the smaller loop is given by

$$e = -\frac{d\varphi}{dt} = -\frac{d\varphi}{dx}\cdot\frac{dx}{dt} = -(1/2)\mu_0 i R^2 \pi r^2 \frac{d}{dx}(R^2+x^2)^{-3/2} v$$

$$= -(1/2)\mu_0 i R^2 \pi r^2 (-3/2)(R^2+x^2)^{-5/2} 2x\, v$$

$$= (3/2)\mu_0 i R^2 \pi r^2 (R^2+x^2)^{-5/2} x v$$

As x >> R we can neglect R^2 as compared to x^2 and hence

$$e = (3/2)\mu_0 i R^2 \pi r^2 x^{-4} v = \frac{3\pi\mu_0 i R^2 r^2 v}{2x^4} \qquad \text{---------- (b)}$$

The sign of induced emf is positive means the induced current is in the same direction as of inducing current. This is evident according to Lenz law. As the loop moves away, the magnetic flux through it decreases, so the induced current is in such direction that it will try to increase the field. For this the field due to induced current should be in the direction of original field and hence the induced current should be in *counterclockwise* direction.